Activity 11 Parametric and vector equations

Aim: Understand the links between different forms of the equations of lines.

From the previous activity *Position vectors and internal division*:

At 2 pm container ship, *Andromeda,* is at a point with position vector [–3, 7] km relative to a port. It is travelling with constant velocity of [12, 9] km/h.

We can express the position of the ship at any time, t hours after 2 pm in the following ways:

vector equation: $\mathbf{r} = [-3,7] + t[12,9]$ or $\mathbf{r} = [-3+12t, 7+9t]$

parametric equations: $\begin{cases} x = -3+12t \\ y = 7+9t \end{cases}$

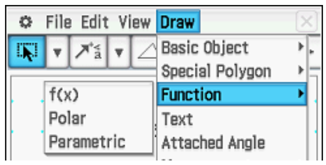
With parametric equations, the x and y values of the ship's co-ordinates are given in terms of a parameter t, where $t \geq 0$.

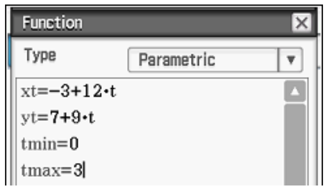
Parametric equations can be graphed in the Geometry application.

Enter parametric equations in Geometry

- Open a new Geometry page with axes and labels shown
- Select [Draw | Function | Parametric]
- Enter the parametric equations

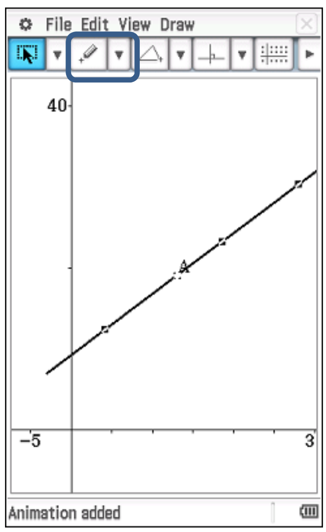
  t is available from the Keyboard Math2 menu
- Set tmax to 3 hours
- Tap OK
- Select [View | Zoom to Fit]





Animate the ship

- Select the draw point tool and tap the path of the ship
- Select the point and the path
- Select [Edit | Animate | Add Animation]
  (A message *Animation added* should appear at the bottom of the screen)
- Select [Edit | Animate | Go (once)] to watch the ship move



A second ship, *Big Dipper*, has a position vector described by the equation $\mathbf{r} = [15-3t,\ 11t+4]$ km, t hours after 2 pm.

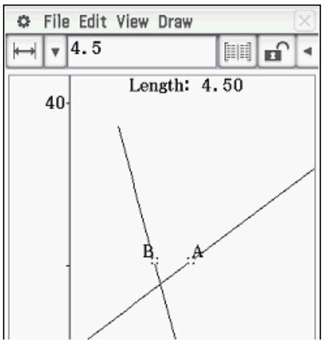
Enter this as a set of parametric equations and add a point on the path to represent the ship. Add a second animation for the *Big Dipper*.

1. Do the ships collide? Run the animation to find out.

The ships must maintain a safe distance of 500m. We can determine the distance between the ships during the animation as follows.

Display distance between the ships

- Tap to select points A and B.
- Tap , the length tool to insert the distance between A and B onto the screen.
- Select [Edit | Animate | Go (once)] to run the animation again.



2. What is the shortest distance you observe during the animation?

The distances you observe are dependent on the animation settings, and your answer to Q2 may not be the minimum distance. We can determine the actual minimum distance using CAS. Carry out the following questions in the Main screen.

3. Determine the separation vector $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$.

4. Do the ships come within 500m of each other?

Learning notes

In the previous activity, Q4 involved a ship with an initial position vector and a constant velocity vector. This information can be combined to describe the location of the ship at any time, *t*, using either parametric or vector equations.

Question 4

The distance is given by $\left|\overrightarrow{AB}\right|$. We require the minimum value of this function. Use fMin to determine the minimum distance between ships A and B and the time at which this minimum occurs.

- Store $\overrightarrow{OA}$
- Store $\overrightarrow{OB}$
- Calculate $\overrightarrow{AB}$
- Calculate $\left|\overrightarrow{AB}\right|$
- Select [Interactive | Calculation | fMin]